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The linear stability in systems with intensive mass transfer—I. Gas (liquid)–solid

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Abstract—A linear analysis of the stability of the flow in a laminar boundary layer under conditions of intensive interphase mass transfer, where high mass fluxes through the phase boundary induce secondary flows, is suggested. These secondary flows have an effect of 'injection' or 'suction' in the boundary layer depending on the direction of the intensive interphase mass transfer, and they have a variable velocity along the length of the phase surface. They are also proportional to the local mass flux through the phase boundary. The critical Reynolds numbers are obtained at different intensities of non-linear mass transfer in the laminar boundary layer. The influence of the direction of the intensive interphase mass transfer on the hydrodynamic stability is shown as well. In the cases shown intensive interphase mass transfer is directed toward the phase boundary (suction) and the increase of its intensity leads to an increase in the stability of the flow. In the opposite case (injection), turbulization results at considerably smaller values of the Reynolds number. Copyright © 1996 Elsevier Science Ltd.

1. INTRODUCTION

A great number of experimental investigations, where the rate of the mass transfer in systems with intensive interphase mass transfer cannot be described using the linear theory of mass transfer, are given in the literature. Very often they are explained with the Marangoni effect and the hydrodynamic instability [1–5], i.e. the induction of secondary flows with tangential direction to the phase boundary. These secondary flows change the shape of the velocity profile of the flow in the boundary layer. Therefore they change the hydrodynamic stability of this flow.

Mass transfer in systems with intensive interphase mass transfer [6] most often is a result of high concentration gradients. As a result of this, high mass fluxes induce secondary flows on the phase boundary [7–9]. The rate of these flows is determined directly from the concentration gradient of the transferred substance through a phase boundary as follows.

$$v = -\frac{MD}{\rho}\frac{\partial c}{\partial n}, \qquad (1)$$

where M is the molecular mass, D is the diffusion coefficient, c is the concentration of transferred substance; ρ is the specific mass at the phase boundary surface and $\partial c/\partial n$ denotes differentiation normal to the interphase.

Equation (1) gives a connection between the velocity of the flow and the concentration distribution which leads to non-linearity in the equation of convective diffusion.

Depending on the direction of interphase mass transfer, this effect of non-linear mass transfer leads to 'injection' or 'suction' in the boundary layer [10], i.e. to a decreasing or increasing of the hydrodynamic stability of the flow.

Analysis of the hydrodynamic stability under these conditions will give an opportunity to distinguish the influence of the Marangoni effect from the effect of non-linear mass transfer on the hydrodynamic stability of the systems with intensive interphase mass transfer.

The theoretical analyses of the hydrodynamics [10: p. 384–386, 45, 71, 98, 100] and the hydrodynamic stability [11] were conducted in the cases, where the normal component of the velocity on the phase boundary is constant along the boundary layer. It is directly seen from equation (1) that under the conditions of non-linear mass transfer in systems with intensive interphase mass transfer the rate of 'suction' ('injection') changes from ∞ to zero. It leads to a change in the velocity profiles in the boundary layer [12]. Hence, it leads to a change of their hydrodynamic stability under these conditions and especially to a change of dependence from the rate and direction of the mass transfer.

2. NON-LINEAR MASS TRANSFER

The intensive interphase mass transfer in the gas (liquid)–solid systems (Fig. 1) will be demonstrated in the case of non-linear mass transfer for a stream

NOMENCLATURE

a	initial value of the Blasius function
A	dimensionless wave number
A_0	parameter
b	initial value of first derivative of the Blasius function
c	concentration
C	dimensionless phase velocity, concentration in non-stationary flow
D	diffusion coefficient
f	Blasius function
F	amplitude
i	imaginary number
k	parameter
L	length
M	molecular mass
p	pressure
P	pressure of non-stationary flow
Re	Reynolds number, $Re = 1.72Re_0$
Sc	Schmidt number
t	time
u	velocity of basic stationary flow in x direction
U	velocity of non-stationary flow in x direction
v	velocity of basic stationary flow in y direction
V	velocity of non-stationary flow in y direction
x	coordinate
y	coordinate.

Greek symbols

α	wavenumber
β/α	phase velocity
γ	parameter
δ	boundary layer thickness
ε	parameter
η	similarity variable
θ	parameter
λ	wavelength
ν	kinematic viscosity
ξ	variable
π	pi
ρ	density
φ	dimensionless amplitude
Φ	stream function in x direction
Ψ	stream function in y direction
ω	small parameter.

Subscripts and superscripts

*	conditions on the solid surface
0	conditions of the transferred substance
1	two-dimensional periodic disturbance
cr	critical number
i	imaginary part of complex number
max	maximum
min	minimum
r	real part of complex number.

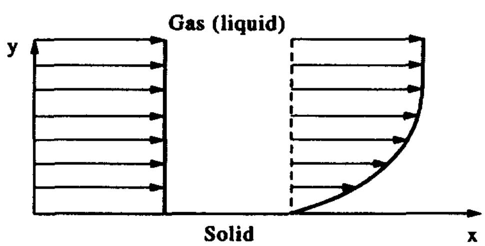

Fig. 1. Velocity profiles in the boundary layer in gas (liquid)–solid systems.

flow along a semi-infinite plate [7, 13, 14, 26]. The mathematical model in this case takes the following form:

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\nu\frac{\partial^2 u}{\partial y^2},$$

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0,$$

$$u\frac{\partial c}{\partial x}+v\frac{\partial c}{\partial y}=D\frac{\partial^2 c}{\partial y^2},$$

$$x=0,\quad u=u_0,\quad c=c_0;$$

$$y=0,\quad u=0,\quad v=-\frac{MD}{\rho_0^*}\frac{\partial c}{\partial y},\quad c=c^*;$$

$$y\to\infty,\quad u=u_0,\quad c=c_0, \tag{2}$$

where ν is viscosity; u_0 is the velocity of potential flow, c_0 is the concentration of transferred substance, c^* is the concentration on the solid surface; ρ_0^* is the gas-(liquid) density on the phase boundary ($y=0$).

The boundary condition for v on a solid surface ($y=0$) leads to non-linearity in the equation that determines c from the system in equation (2) (in the approximation of the linear theory of mass transfer this condition has a form $v=0$).

The solution of problem (2) can be obtained, if the following similarity variables are used:

$$u=0.5u_0\varepsilon\Phi',\quad v=0.5\left(\frac{u_0\nu}{x}\right)^{0.5}(\eta\Phi'-\Phi),$$

$$c=c_0+(c^*-c_0)\Psi,\quad \Phi=\Phi(\eta),\quad \Psi=\Psi(\eta),$$

$$\eta = \left(\frac{u_0}{4Dx}\right)^{0.5}, \quad \varepsilon = Sc^{0.5}, \quad Sc = \frac{\nu}{D}, \tag{3}$$

where Sc is the Schmidt number.

Substitution of equation (3) into system (2) leads to a system of ordinary differential equations

$$\Phi''' + \varepsilon^{-1}\Phi\Phi'' = 0, \quad \Psi'' + \varepsilon\Phi\Psi' = 0,$$

$$\Phi(0) = \theta\Psi'(0), \quad \Phi'(0) = 0, \quad \Phi'(\infty) = 2\varepsilon^{-1},$$

$$\Psi(0) = 1, \quad \Psi(\infty) = 0, \tag{4}$$

where θ is a small parameter, which characterizes the non-linearity of mass transfer and depends on the intensity of the interphase mass transfer:

$$\theta = \frac{M(c^* - c_0)}{\varepsilon\rho_0^*}. \tag{5}$$

Problem (4) has been solved [13–15] numerically and asymptotically as well. The results obtained by asymptotic theory [13] are confirmed using direct numerical experiments [13, 15] and show that the secondary flow with the rate $\Phi(0) = \theta\Psi'(0)$ does not change the character of the flow in the boundary layer, but only the shape of the velocity profile $\Phi(\eta)$ [14]. It is also proved by the following theoretical evaluations. The induction of secondary flows [equation (1)] on the face boundary has an effect of injection in (suction from) the boundary layer, depending on the direction of interphase mass transfer. This effect affects the potentiality of the flow at $y \to \infty$ and is not in contradiction with the boundary layer approximations used [10]

$$v_0 < u_0 Re_L^{-1/2}, \quad Re_L = \frac{u_0 L}{\nu}, \tag{6}$$

where v_0 is the mean rate of injection (suction) through the solid surface of length L

$$v_0 = \frac{1}{L}\int_0^L v\,dx, \quad v = \frac{MD}{\rho_0^*}\left(\frac{\partial c}{\partial y}\right)_{y=0}. \tag{7}$$

Introducing equation (3) into equation (8) leads to the following:

$$v_0 = -\theta u_0 Re_L^{-1/2}\Psi'(0). \tag{8}$$

Comparison of (7) with equation (9) shows that equation (7) is valid when

$$|\theta\Psi'(0)| < 1. \tag{9}$$

Taking into account that $|\Psi'(0)| < 1$, obviously at $|\theta| < 1$, the condition (7) is always valid.

Analytical and numerical solutions of the problem (4) for different values of ε and θ allow the initial values of Φ and its derivatives to be found

$$\Phi(0) = a, \quad \Phi'(0) = 0, \quad \Phi''(0) = b, \tag{10}$$

Table 1. Initial values of Φ, its derivatives and parameter k

ε	θ	a	b	k
1	−0.30	0.2546	1.710	1.232
	−0.20	0.1557	1.557	1.414
	−0.10	0.07162	1.432	1.576
	0	0	1.329	1.718
	0.10	−0.06196	1.239	1.849
	0.20	−0.1162	1.162	1.968
	0.30	−0.1643	1.095	2.076
10	−0.05	0.02295	0.01359	1.673
	0	0	0.01328	1.718
	0.05	−0.01237	0.01309	1.745
	0.10	−0.02074	0.01298	1.763
	0.20	−0.03196	0.01281	1.786
20	−0.05	0.02395	0.003389	1.668
	−0.03	0.01219	0.003375	1.697
	0	0	0.003321	1.718
	0.03	−0.00570	0.003321	1.734

and some of its values [13] are shown in Table 1. The linear analysis is made considerably easier considering equation (4) as a Cauchy problem.

It is seen from Table 1 that the initial conditions a and b include the effect of the mass transfer on the velocity profiles in the boundary layer and depend considerably on the magnitude and the direction of the rate of the induced flow, respectively, on the rate and the direction of the intensive interphase mass transfer.

At high values of θ, in the case of liquids ($\varepsilon \gg 1$), numerical solution cannot converge due to an increasing singular perturbation (or stiffness) of the solution of the boundary layer.

It is seen from Table 1 that $\theta > 0$ ($\theta < 0$) corresponds to 'injection' in ('suction' from) the boundary layer and according to the theory of the hydrodynamic stability [10] the decrease (increase) of the hydrodynamic stability of the flow in the boundary layer should be expected.

3. STABILITY

The influence of intensive interphase mass transfer on the hydrodynamic stability of the flows in a laminar boundary layer will be investigated by applying the linear stability theory [10, 16]. This theory will be applied also for an almost parallel flow in a boundary layer, as it has been done in refs. [17, 18], taking into account two linear scales

$$x, \quad \delta = \sqrt{\frac{\nu x}{u_0}}. \tag{11}$$

The relation between these two scales for $x = L$ is connected with the Reynolds number

$$Re_L = \frac{u_0 L}{\nu} = \left(\frac{L}{\delta}\right)^2 \gg 1. \tag{12}$$

The approximations of the boundary layer [equation (2)] are zero-th order approximation in regard of the small parameters $(\delta/L)^2$, i.e. we use substantially the following relations:

$$\frac{\partial^2 v}{\partial x \partial y} \approx \frac{\partial^2 u}{\partial x^2} \ll \frac{\partial^2 u}{\partial y^2}, \quad \frac{\partial^2 v}{\partial x^2} \ll \frac{\partial^2 v}{\partial y^2}, \tag{13}$$

they will be used in our analysis.

The linear stability analysis considers a non-stationary flow (U, V, P), obtained as a combination of a basic stationary flow (u, v) and two-dimensional periodic disturbances (u_1, v_1, p_1) with small amplitudes $(\omega \ll 1)$:

$$\begin{aligned} U(x,y,t) &= u(x,y)+\omega \cdot u_1(x,y,t), \\ V(x,y,t) &= v(x,y)+\omega \cdot v_1(x,y,t), \\ P(x,y,t) &= \omega \cdot p_1(x,y,t), \\ C(x,y,t) &= c(x,y)+\omega \cdot c_1(x,y,t). \end{aligned} \tag{14}$$

The non-stationary flow, thus obtained, satisfies the full system of Navier–Stokes equations

$$\begin{aligned} &\frac{\partial U}{\partial t}+U\frac{\partial U}{\partial x}+V\frac{\partial U}{\partial y} = -\frac{1}{\rho}\frac{\partial P}{\partial x}+\nu\left(\frac{\partial^2 U}{\partial x^2}+\frac{\partial^2 U}{\partial y^2}\right), \\ &\frac{\partial V}{\partial t}+U\frac{\partial V}{\partial x}+V\frac{\partial V}{\partial y} = -\frac{1}{\rho}\frac{\partial P}{\partial y}+\nu\left(\frac{\partial^2 V}{\partial x^2}+\frac{\partial^2 V}{\partial y^2}\right), \\ &\frac{\partial U}{\partial x}+\frac{\partial V}{\partial y} = 0, \\ &\frac{\partial C}{\partial t}+U\frac{\partial C}{\partial x}+V\frac{\partial C}{\partial y} = D\left(\frac{\partial^2 C}{\partial x^2}+\frac{\partial^2 C}{\partial y^2}\right), \\ &x = 0, \quad U = u_0, \quad V = 0, \quad P = p_0; \\ &y = 0, \quad U = 0, \quad V = -\theta A_0\frac{\partial C}{\partial y}; \\ &y \to \infty, \quad U = u_0, \quad V = 0, \quad P = p_0, \end{aligned} \tag{15}$$

where A_0

$$A_0 = \frac{\varepsilon D}{c^* - c_0}. \tag{16}$$

After linearizing equation (15), i.e. in zero approximation of the small parameters ω^2 and $\theta \cdot \omega$, substitution of equation (13) and (14) in equation (15) leads to the following problem:

$$\begin{aligned} &\frac{\partial u_1}{\partial t}+u\frac{\partial u_1}{\partial x}+v\frac{\partial u_1}{\partial y}+u_1\frac{\partial u}{\partial x}+v_1\frac{\partial u}{\partial y} \\ &\qquad = -\frac{1}{\rho}\frac{\partial p_1}{\partial x}+\nu\left(\frac{\partial^2 u_1}{\partial x^2}+\frac{\partial^2 u_1}{\partial y^2}\right), \\ &\frac{\partial v_1}{\partial t}+u\frac{\partial v_1}{\partial x}+v\frac{\partial v_1}{\partial y}+u_1\frac{\partial v}{\partial x}+v_1\frac{\partial v}{\partial y} \\ &\qquad = -\frac{1}{\rho}\frac{\partial p_1}{\partial y}+\nu\left(\frac{\partial^2 v_1}{\partial x^2}+\frac{\partial^2 v_1}{\partial y^2}\right), \\ &\frac{\partial u_1}{\partial x}+\frac{\partial v_1}{\partial y} = 0; \\ &x = 0, \quad u_1 = 0, \quad v_1 = 0, \quad p_1 = p_0; \\ &y = 0, \quad u_1 = 0, \quad v_1 = 0, \quad p_1 = p_0; \\ &y \to \infty, \quad u_1 = 0, \quad v_1 = 0. \end{aligned} \tag{17}$$

Differentiating between y and x of the first two equations gives us the opportunity to exclude the pressure p_1. The stability of the basic flow will be examined considering periodic disturbances of the form

$$\begin{aligned} u_1 &= F'(y)\exp i(\alpha x-\beta t), \\ v_1 &= -i\alpha F(y)\exp i(\alpha x-\beta t), \end{aligned} \tag{18}$$

where $F(y)$ is the amplitude of one-dimensional disturbance (regarding y); α and β/α are, respectively, its wavenumber and phase velocity

$$\alpha = \frac{2\pi}{\lambda}, \quad \beta = \beta_r + i\beta_i. \tag{19}$$

In expression (19), λ is the wavelength, β_r is the circle frequency and β_i is the increment factor. Obviously, the condition for the stability of the flow is

$$\beta_i < 0. \tag{20}$$

In the case of $\beta_i > 0$ the basic flow is unstable (the amplitude grows with time).

4. GOVERNING EQUATION

Introducing equation (18) into equation (17) leads to Orr–Sommerfeld type of equations [25, 27] for the amplitude of the disturbances

$$\begin{aligned} &\left(u-\frac{\beta}{\alpha}\right)(F''-\alpha^2 F)-\frac{\partial^2 u}{\partial y^2}F \\ &\qquad = -\frac{i\nu}{\alpha}(F^{\text{iv}}-2\alpha^2 F''+\alpha^4 F) \\ &\qquad\quad +\frac{i}{\alpha}\left[vF'''+\left(\frac{\partial^2 u}{\partial x \partial y}-\alpha^2 v\right)F'\right], \\ &y = 0, \quad F = 0, \quad F' = 0, \\ &y \to \infty, \quad F = 0, \quad F' = 0. \end{aligned} \tag{21}$$

In equation (21) we have $F = F(y)$, while u and v depend on y, and νx, hence, the dependence on x is insufficient. This gives us an opportunity to consider x as a parameter [19]. There are four constants in equation (21), where ν and α are known beforehand, while eigenvalues of β_r and β_i as well as eigenfunction $F(y)$, are looked for. Obviously thus determined eigenvalues of β_r and β_i depend on x and at some x_{cr}

$$\beta_i(x_{cr}) = 0, \tag{22}$$

i.e. the velocity profile $u(x, y)$ is not stable.

The assumption that the variable x is a parameter in equation (21) allows a new variable to be introduced

$$\xi = \frac{y}{\delta} = y\left(\frac{u_0}{\nu x}\right)^{0.5} = \frac{2}{\varepsilon}\eta. \tag{23}$$

Hence all functions in equation (21) can be expressed by the new variable ξ (23)

$$u = u_0 f'(\xi), \quad v = 0.5\left(\frac{u_0 \nu}{x}\right)^{0.5}(\xi f' - f),$$

$$F(y) = \varphi(\xi), \quad F^{(j)} = \delta^{-j}\varphi^{(j)}, \quad j = 1, \ldots, 4. \tag{24}$$

It is seen from equations (4), (6) and (24) that f can be determined from

$$2f''' + ff'' = 0,$$

$$f(0) = a, \quad f'(0) = 0, \quad f''(0) = \frac{\varepsilon^2}{4}b. \tag{25}$$

Introducing equation (24) in equation (21) leads to the following Orr–Sommerfeld type of equation:

$$(f' - C)(\varphi'' - A^2\varphi) - f'''\varphi$$
$$= -\frac{i}{A\,Re_0}\left\{(\varphi^{iv} - 2A^2\varphi'' + A^4\varphi) - \tfrac{1}{2}(\xi f' - f)\varphi'''\right.$$
$$\left. + \left[\frac{1}{2}(\xi f''' + f'') + \frac{A^2}{2}(\xi f' - f)\right]\varphi'\right\},$$

$$\varphi(0) = 0, \quad \varphi'(0) = 0, \quad \varphi(\infty) = 0, \quad \varphi'(\infty) = 0, \tag{26}$$

where

$$A = \alpha\delta, \quad C = \frac{\beta}{\alpha u_0} = C_r + iC_i, \quad Re = 1.72\frac{u_0\,\delta}{\nu}. \tag{27}$$

The linear analysis of the hydrodynamic stability of a laminar boundary layer under the conditions of intensive interphase mass transfer finally are reduced to determining C_r and $\varphi(\xi)$ at $C_i = 0$, when Re and A are given. The minimum Reynolds number, i.e. the critical Reynolds number Re_{cr}, when the flow ceases to be stable, can be obtained from the dependence $C_r(Re)$.

5. NUMERICAL METHOD

The problem (26) is an eigenvalue problem about C, when Re and A are given. The imaginary part of the eigenvalue C determines whether or not the basic flow is stable relative to infinitesimal disturbances. Since the problem is a linear eigenvalue problem, in theory it is possible to solve for $C = C(Re, A)$. Solutions for this problem are usually presented in two ways: for specific values of the parameters A and Re, the corresponding value of C is tabulated or, the locus in the (Re, A) plane of which $C_i = 0$ (the 'neutral stability curve') is plotted. The critical Reynolds number is the minimum Reynolds number for which an infinitesimal disturbance will grow. We apply the so called time growing disturbances when Re and A are given real values, whereas the parameter C is the searching complex eigenvalue.

In order to solve problem (26) on the infinite interval numerically, the boundary conditions ($\varphi(\infty) = 0$, $\varphi'(\infty) = 0$) are assumed to be at a finite distance $\xi = \xi_\infty \gg 1$ far from the plate. Far from the wall, the boundary conditions will be replaced by two differential equations. In order to gain these equations the solution [10] of equation (25) is used at higher values of ξ

$$f(\xi) = \xi - k + 0.231\int_\infty^\xi \mathrm{d}\xi \int_\infty^\xi \exp\left[-\tfrac{1}{4}(\xi - k)^2\right]\mathrm{d}\xi. \tag{28}$$

The numerical solution of equation (25) and its comparison with equation (28) shows that an accuracy of $10^{-4} - 10^{-6}$ is reached when ξ is greater than 5–6, and we can assume

$$f' = 1, \quad f'' = f''' = 0, \quad \xi f' - f = k, \quad \xi f''' = 0. \tag{29}$$

Thus introducing equation (29) into equation (26) leads to the expression, which is valid in the case of $\xi \geqslant 6$

$$(1 - C)(\varphi'' - A^2\varphi) = -\frac{i}{A\,Re}\left[(\varphi^{iv} - 2A^2\varphi'' + A^4\varphi) - \tfrac{1}{2}k\varphi''' + \frac{A^2}{2}k\varphi'\right]. \tag{30}$$

The solution of equation (30) depends on four constants [20, 21], two of them are equal to zero, because two of the solutions of the characteristic equation (30) are positive, i.e. they satisfy conditions $\varphi(\infty) = \varphi'(\infty) = 0$:

$$\varphi = C_1\exp(-A\xi) + C_2\exp(-\gamma\xi), \tag{31}$$

where the constants C_1 and C_2 are determined using the boundary conditions. If we exclude these constants from equation (31) this will lead to the following relations for $\xi \geqslant \xi_\infty = 6$:

$$(\varphi'' - A^2\varphi)' - \gamma(\varphi'' - A^2\varphi) = 0, \quad \xi = \xi_\infty,$$
$$(\varphi'' - \gamma^2\varphi)' + A(\varphi'' - \gamma^2\varphi) = 0, \tag{32}$$

where we obtain for γ the following

$$\gamma = \frac{k}{4} - \frac{\sqrt{k^2+16A[A+i\,Re(1-C)]}}{4}. \qquad (33)$$

The numerical solution of equation (25) for different values of θ shows that k depends on θ (Table 1). In the case of $\theta = 0$ the comparison of $Re_{cr} \approx 500$ [22–23] in the approximation of the parallel flows with $Re_{cr} = 501$ obtained by us in a case of almost parallel flows, shows that Re_{cr} depends slightly on k. Analogous results have been obtained at $\theta \neq 0$.

In matrix form equation (26) gains the following form:

$$\begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \end{bmatrix}' + \begin{bmatrix} 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \\ a_1 & a_2 & a_3 & a_4 \end{bmatrix} \begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ b_1 \end{bmatrix} = 0, \qquad (34)$$

and a_j $(j = 1, \ldots, 4)$ are obtained directly from equation (26):

$$a_1 = [iA^3\,Re(f'-B) - iA\,Ref''' + A^4],$$

$$a_2 = \tfrac{1}{2}(\xi f''' - f'') + \frac{A^2}{2}(\xi f' - f),$$

$$a_3 = -[iA\,Re(f'-B) + 2A^2],$$

$$a_4 = -\tfrac{1}{2}(\xi f' - f),$$

where b_j $(j = 1, \ldots, 4)$

$$b_1 = \varphi, \quad b_2 = \varphi', \quad b_3 = \varphi'', \quad b_4 = \varphi'''.$$

The boundary conditions are transformed in

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \end{bmatrix} = 0, \quad \xi = 0; \qquad (35)$$

and

$$\begin{bmatrix} -\gamma A^2 & -A^2 & \gamma & 1 \\ -A\gamma^2 & -\gamma^2 & A & 1 \end{bmatrix} \begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \end{bmatrix} = 0, \quad \xi = \xi_\infty = 6, \qquad (36)$$

respectively.

Using substitution

$$b_j = \varphi^{(j)}(\xi), \quad (j = 1, \ldots, 4), \quad \mathbf{B} = (b_1, b_2, b_3, b_4)^{\mathrm{T}}.$$

The eigenvalue problem, [equations (35) and (36)] can be written in the form

$$\mathbf{B}'(\xi) + \mathbf{A}(\xi; C)\mathbf{B}(\xi) = 0, \quad \xi \in [0, \xi_\infty]; \qquad (37)$$

$$\mathbf{\Psi}_0^{\mathrm{T}}\mathbf{B} = 0, \quad \xi = 0; \quad \mathbf{\Psi}_1^{\mathrm{T}}\mathbf{B} = 0, \quad \xi = \xi_\infty, \qquad (38)$$

where $\mathbf{A}(\xi; C)$ is 4×4-matrix of continuous components about $\xi \in [0, \infty]$ and depending on c; $\mathbf{\Psi}^{\mathrm{T}}$ and $\mathbf{\Psi}_0^{\mathrm{T}}$ are scalar matrices of order 4×2 ($\mathbf{\Psi}^{\mathrm{T}}$ denotes a transpose matrix of $\mathbf{\Psi}$).

To solve the eigenvalue problem, (37) and (38) we use the method proposed by Abramov [24]. Let $\mathbf{B}(\xi; C)$ be an arbitrary solution of system (37) satisfying the boundary condition at $\xi = \xi_\infty$. Then, as it has been shown in ref. [24], the solution $\mathbf{\Psi}(\xi; C)$ of the initial value problem

$$\mathbf{\Psi}' - (\mathbf{A}^{\mathrm{T}} + \mathbf{\Psi}(\mathbf{\Psi}^{\mathrm{T}}\mathbf{\Psi})^{-1}\mathbf{\Psi}^{\mathrm{T}}\mathbf{A}^{\mathrm{T}})\mathbf{\Psi} = 0, \quad \xi \in [0, \infty], \qquad (39)$$

$$\mathbf{\Psi} = \mathbf{\Psi}_1, \quad \xi = \xi_\infty, \qquad (40)$$

satisfies

$$\mathbf{\Psi}(\xi; C)\mathbf{B}(\xi; C) = 0 \quad \text{for any} \quad \xi \in [0, \infty],$$

i.e. we can have the boundary conditions at $\xi = \xi_\infty$ transferred to any $\xi \in [0, \infty]$.

Hence, by integrating equations (39) and (40) up to $\xi = 0$ the required eigenvalue relation is obtained in the form

$$\det \begin{pmatrix} \mathbf{\Psi}_0^{\mathrm{T}} \\ \mathbf{\Psi}_{1,0}^{\mathrm{T}}(C) \end{pmatrix} = 0, \qquad (41)$$

where $\mathbf{\Psi}_{1,0}(C)$ denotes the solution of equations (39) and (40) at $\xi = 0$.

The proposed method is stable and $\mathbf{\Psi}\mathbf{\Psi}^{\mathrm{T}} = \text{const}$ along the integration path. The basic procedure is to iterate C until the solution C^* of characteristic equation (41) is obtained with a given accuracy. The same procedure has to be repeated with greater ξ_∞ with a view to convergence of the successive approximations C^*. When a convergence is established with the prescribed accuracy the last computer C^* is taken as an eigenvalue of the original problem (26). The numerical experiments show that an accuracy of $10^{-4}-10^{-6}$ is reached when ξ_∞ is greater than 5–6.

6. RESULTS AND DISCUSSION

The neutral curves presented in (Re, A) as well as in the (Re, C) plane are given in Figs 2–7. They are obtained for gases ($\varepsilon = 1$) and for liquids ($\varepsilon = 10, 20$) as well.

The critical Reynolds numbers Re_{cr}, corresponding wave velocities C_r and wave numbers A are obtained. $C_{r\,min}$ and A_{min} are also obtained from these results. We denote $C_{r\,min}$ and A_{min} the minimal values for wave velocities and wave number at which the flow is stable at any Reynolds number Re, respectively. They are shown in Table 2 with dependence on the magnitude and on the direction of the concentration gradient, at the conditions of an intensive interphase mass transfer.

It is seen from Figs 2–7 and from Table 2, that the intensive interphase mass transfer directed toward the

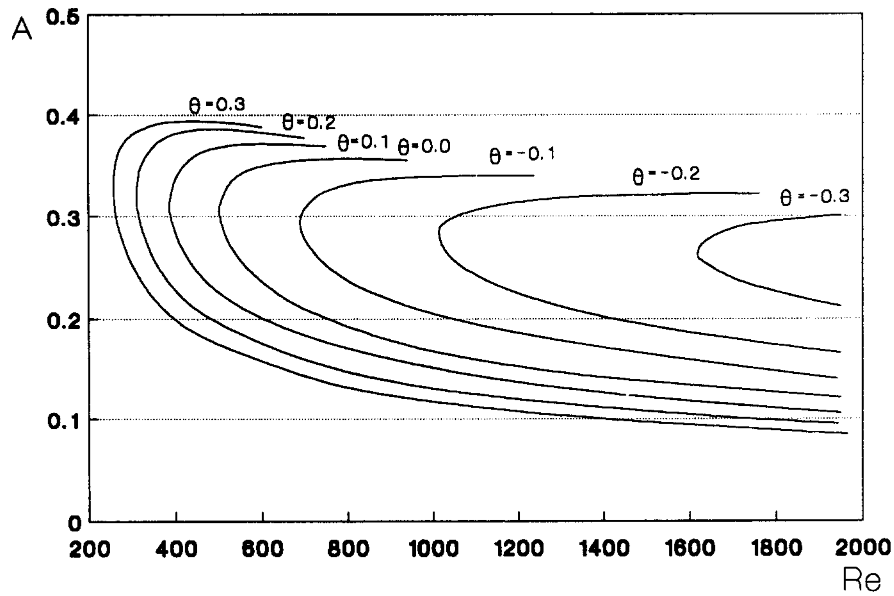

Fig. 2. The neutral curve for the wave number A as a function of the Reynolds number Re in a case of $\varepsilon = 1$.

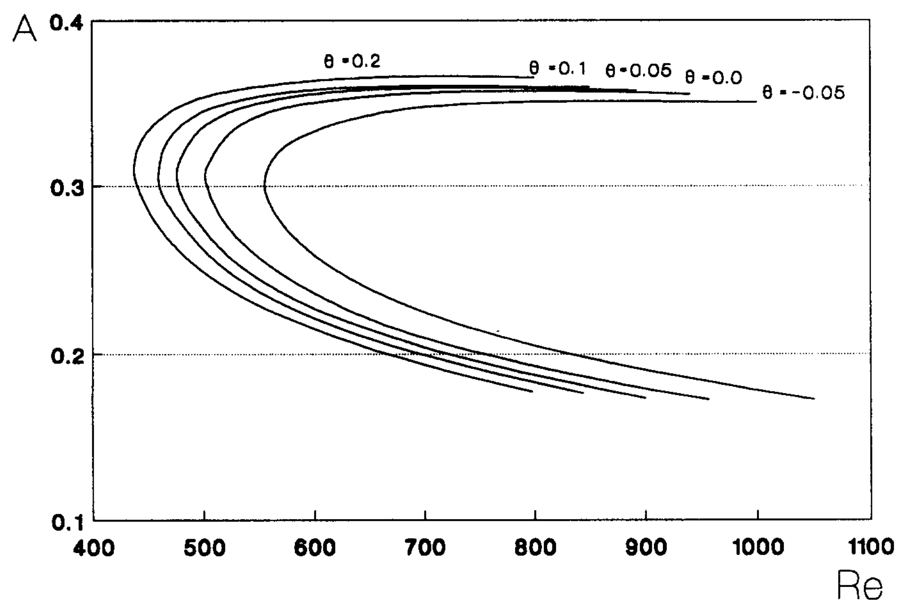

Fig. 3. The neutral curve for the wave number A as a function of the Reynolds number Re in a case of $\varepsilon = 10$.

phase boundary ($\theta < 0$) (the effect of 'suction') stabilizes the flow, i.e. the rise of the concentration difference $|c_0 - c^*|$ leads to an increase of Re_{cr} and to a decrease of $C_{r\,min}$ and A_{min}. In the case of intensive interphase mass transfer directed from the phase boundary toward the volume ($\theta > 0$) (the effect of 'injection') a destabilization of the flow is observed, i.e. the rise of the concentration difference $|c_0 - c^*|$ leads to a decrease of Re_{cr} and to an increase of $C_{r\,min}$ and A_{min}.

The high concentration gradients have a stabilizing effect at $\theta < 0$, that is significantly higher than the destabilizing one in the case of a change in the direction of mass transfer ($\theta > 0$).

The discussions above are taking into account the fact that diffusive fluxes through the face boundary at ($\theta < 0$) increase with the rise of the concentration difference $|c_0 - c^*|$, while at ($\theta > 0$) they decrease with the rise of $|c^* - c_0|$.

The results obtained could be of use in clarification of the mechanism and the kinetics of a number of practically interesting processes. For instance, in liquid–solid systems the anode dissolving of metals in the electrolyte flow under the conditions of intensive

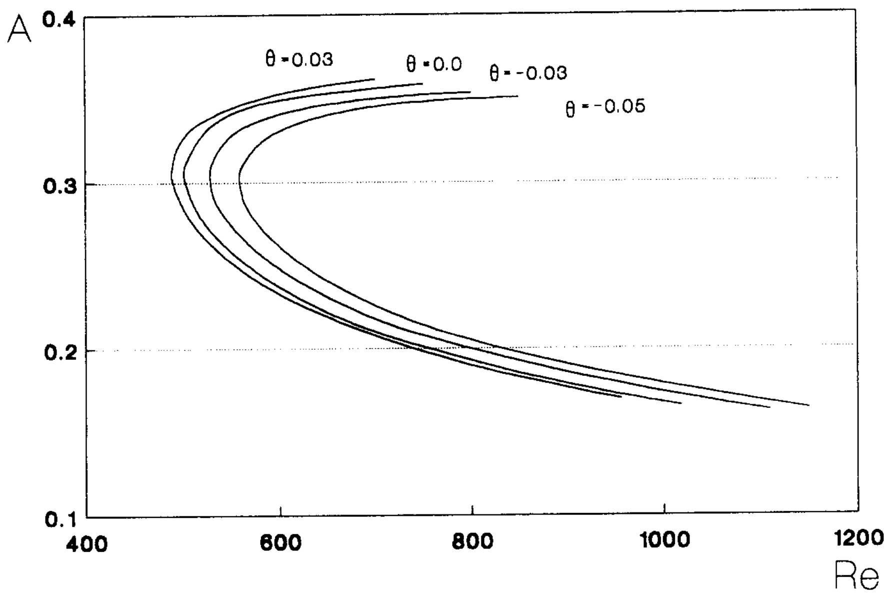

Fig. 4. The neutral curve for the wave number A as a function of the Reynolds number Re in a case of $\varepsilon = 20$.

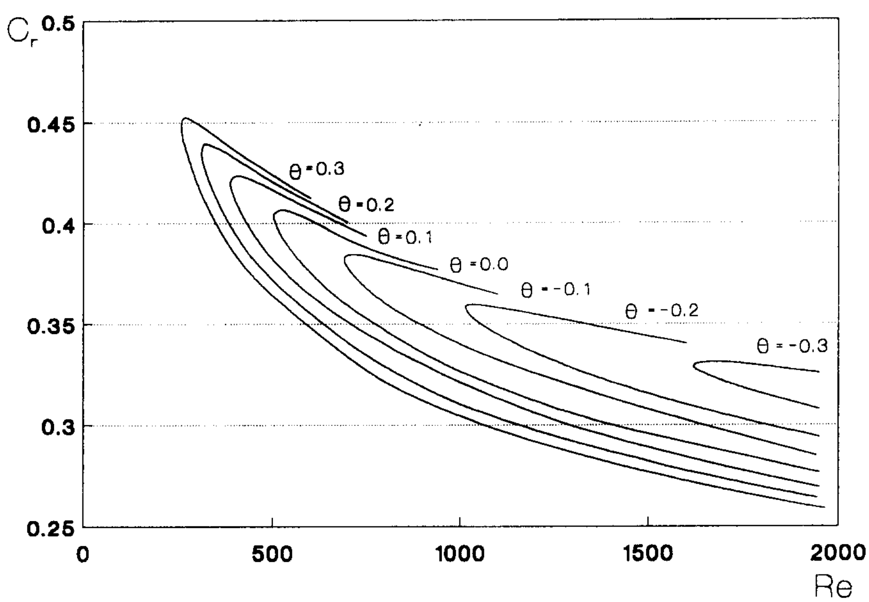

Fig. 5. The neutral curve for the wave velocities C_r as a function of the Reynolds number Re in a case of $\varepsilon = 1$.

interphase mass transfer can rise substantially before flow turbulence for comparatively small values of Reynolds number, while the electrode position of metals out of concentrated solutions can be implemented at the laminar conditions at high values of Reynolds number. The intensive interphase mass transfer is of interest for the process of ablation (for example, launching a spacecraft in a denser atmospheric layer). Intensive evaporation of substance from a solid surface leads to an increase of the interphase heat transfer coefficients, i.e. to a decrease of 'undesired' heat flux toward the spacecraft (missile) rounded fuselage nose. It is seen from the results obtained that at these conditions the turbilization of gas at considerably small

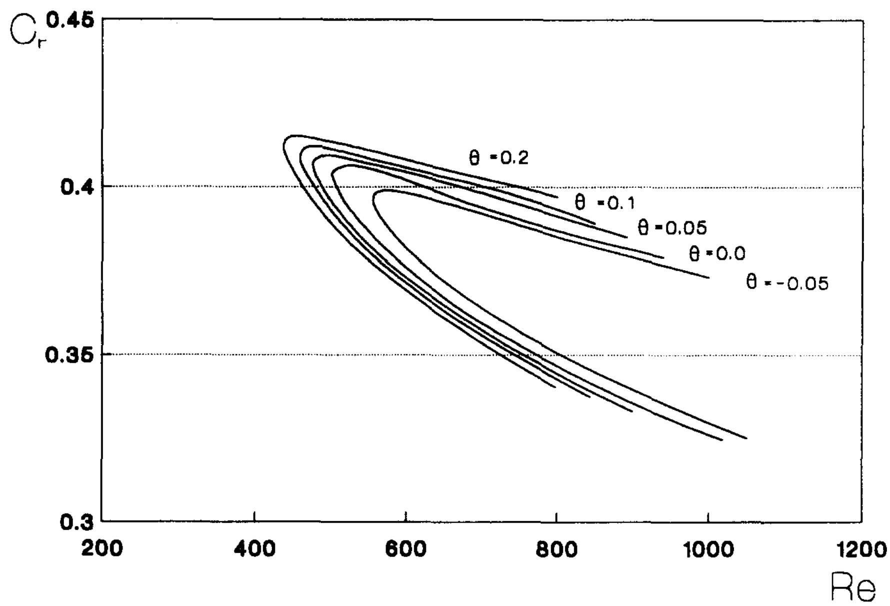

Fig. 6. The neutral curve for the wave velocities C_r as a function of the Reynolds number Re in a case of $\varepsilon = 10$.

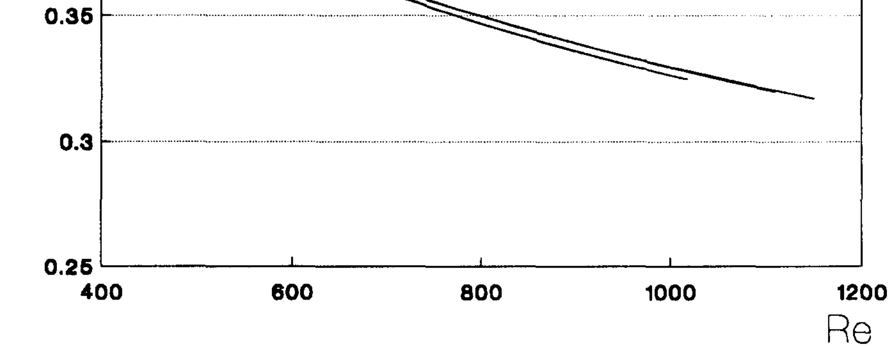

Fig. 7. The neutral curve for the wave velocities C_r as a function of the Reynolds number Re in a case of $\varepsilon = 20$.

Reynolds numbers is possible, which will affect also the rate of interphase heat transfer.

The observed influence of the intensive interphase mass transfer on the hydrodynamic stability in gas (liquid)–solid systems is much more interesting for systems with a movable face boundary (gas–liquid, liquid–liquid), which will be discussed in a future paper.

Table 2. Values of the critical Reynolds numbers Re_{cr}, corresponding wave velocities C_r, wave numbers A and $C_{r\,min}$, A_{min} obtained

ε	θ	Re_{cr}	A	C_r	A_{min}	$C_{r\,min}$
1	−0.30	1619	0.259	0.3281	0.301	0.3310
	−0.20	1014	0.285	0.3587	0.322	0.3599
	−0.10	689	0.290	0.3816	0.340	0.3848
	0	501	0.305	0.4035	0.359	0.4067
	0.10	386	0.309	0.4196	0.373	0.4243
	0.20	310	0.320	0.4351	0.387	0.4396
	0.30	258	0.331	0.4488	0.398	0.4526
10	−0.05	555	0.300	0.3960	0.351	0.3990
	0	501	0.305	0.4035	0.359	0.4067
	0.05	476	0.305	0.4062	0.360	0.4097
	0.10	459	0.305	0.4085	0.361	0.4124
	0.20	437	0.310	0.4123	0.367	0.4155
20	−0.05	558	0.305	0.3959	0.351	0.3978
	−0.03	528	0.305	0.4010	0.354	0.4037
	0	501	0.305	0.4035	0.359	0.4067
	0.03	488	0.305	0.4064	0.362	0.4099

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